1. Complex numbers come in pairs called conjugates. For example, the conjugate of

3 – 5i is 3 + 5i.

Simplify these expressions involving conjugates:

a) (3 – 5i) + (3 + 5i)

b) (3 + 5i) – (3 – 5i)

c) (3 + 5i)(3 – 5i)

2. Consider the general complex number a + bi.

a) What is its conjugate?

b) Show that when you add a complex number and its conjugate, the result is a real number.

c) Show that when you multiply a complex number and its conjugate, the result is a real number.

d) When is the sum of a number and its conjugate 0?

e) When is the product of a number and its conjugate 0?

3. Find the complex number a + bi that satisfies:

(a + bi)(1 + i) = 11 – 3i

4. a) Write each expression in factored form:

x2 – 4 x2 – 64 x2 – 100 x2 – 81 x2 + 4

b) Show that x2 + 1 = (x – i)(x + i).

c) You may have noticed that x2 + 4 from part (a) “can’t” be factored. How could you use part (b) to factor x2 + 4?

d) How can this force us to “refine” our definition of factoring?

5. Find all complex numbers a + bi that satisfy:

(a + bi)2 = -11 + 60i

6. a) Use the quadratic formula to find the solutions to the function

f(x) = x2 – 10x + 34.

b) Find the sum and product of the solutions. What do you notice?

c) Write the factored form of this function and show how it is equivalent to

f(x) = x2 – 10x + 34.

7. Find two complex numbers with a sum of 14 and a product of 74.

8. a) What is the conjugate of 2i? b) What is the conjugate of 7?

9. Let z = a + bi and w = c + di. The expression represents the conjugate of a + bi, and the expression represents the conjugate of c + di.

a) Prove that .

b) Prove that .

c) Show that z + = 2a. d) Show that z = a2 + b2.